

KNOW	APPS	THINK	COMM	TOTAL
/12	/14	/10	/8	/44

Read every question and instruction carefully. All work must be shown to receive full marks. All answers should be rationalized, simplified, and written in exact forms, unless otherwise stated. Where necessary, round answers to 4 decimal places.

PART A – Fill in the blanks.

1) Write an equivalent expression for the following in radian exact form:

[K3]

a) $\sin \frac{\pi}{5} = \cos \frac{3\pi}{10}$ b) $2\cos^2\left(\frac{\pi}{14}\right) - 1 = \cos \frac{\pi}{7}$ c) $\frac{1}{4}\sin \frac{1}{3}\theta \cos \frac{1}{3}\theta = \frac{1}{8}\sin\left(\frac{2}{3}\theta\right)$

$$\frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi - 2\pi}{10}$$

$$\frac{\pi \cdot 2}{14} = \frac{\pi}{7}$$

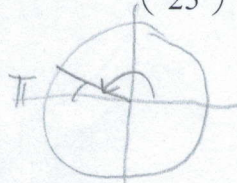
$$2 \cdot \frac{1}{4} = \frac{1}{2}; \sin 2x = 2\sin x \cos x$$

2) Express each as a function of the related acute angle, where $0 \leq \theta \leq 2\pi$.

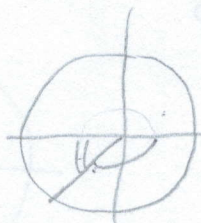
Include a well-labelled diagram.

[K2,C2]

a) $\sin\left(\frac{21\pi}{25}\right) = \sin\left(\frac{25\pi - 4\pi}{25}\right)$



$$= \sin \frac{4\pi}{25}$$



b) $\tan\left(-\frac{7\pi}{8}\right) = \tan\left(\frac{9\pi}{8}\right)$

$$\tan \frac{9\pi - 8\pi}{8} = \tan \frac{\pi}{8}$$

PART B – Give full solution. Show all your work.

3) Write an equivalent expression.

[K3,T3]

a) $\sin(2\pi - x) = \sin(-x) = -\sin(x)$

b) $\tan(-x) = -\tan x$

c) $\cos\left(\frac{\pi}{2} + x\right) = \cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x = -\sin x$

4) Determine the **exact value** of:

a) $\cos 15^\circ$

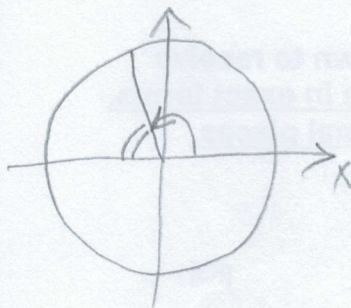
[K2]

$$\begin{aligned} &= \cos(45^\circ - 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \end{aligned}$$

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b) $\cot \frac{7\pi}{12}$; $\tan \frac{7\pi}{12} = \tan \left(\frac{\pi}{3} + \frac{\pi}{4} \right)$

[K2, C2]



$$\begin{aligned} &= \frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{4}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \\ \cot \frac{7\pi}{12} &= \frac{(1 - \sqrt{3})(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = -\frac{3 + 1 - 2\sqrt{3}}{3 - 1} \\ &= -\frac{4 - 2\sqrt{3}}{2} = -(2 - \sqrt{3}) = \sqrt{3} - 2 \end{aligned}$$

5) Solve for x where $0 \leq x \leq 2\pi$

a) $4 \sec x + 2 = 7 + 2 \sec x$

[A3]

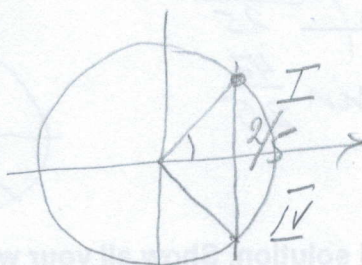
$$4 \sec x - 2 \sec x = 7 - 2$$

$$2 \sec x = 5$$

$$\sec x = \frac{5}{2}$$

$$\cos x = \frac{2}{5}$$

$$\begin{aligned} x_1 &= \cos^{-1}\left(\frac{2}{5}\right) \doteq 1.1071 \text{ (Q I)} \\ x_2 &= 2\pi - 1.1071 \doteq 5.1761 \text{ (Q IV)} \end{aligned} \quad \left. \begin{array}{l} \text{Q I} \\ \text{Q IV} \end{array} \right\} x = 1.1071, 5.1761$$



b) $\sin x - \sqrt{3} \sin x \cos x = 0$

[A3]

$$\sin x (1 - \sqrt{3} \cos x) = 0$$

$$\sin x = 0$$

$$x = 0, \pi, 2\pi \checkmark$$

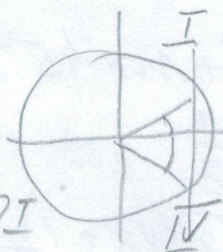
$$1 - \sqrt{3} \cos x = 0$$

$$\cos x = \frac{1}{\sqrt{3}}$$

$$x = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \doteq 0.9553 \text{ (Q I)}$$

$$x_1 \doteq 0.9553$$

$$x_2 = 2\pi - 0.9553 \doteq 5.3279 \text{ (Q IV)}$$



$$\boxed{x = 0, \pi, 2\pi, 0.9553, 5.3279}$$



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c) $6\cos^2 x - 2 = -5\sin x$

[A4]

$$6(1 - \sin^2 x) - 2 + 5\sin x = 0$$

$$-6\sin^2 x + 5\sin x + 4 = 0$$

$$6\sin^2 x - 5\sin x - 4 = 0$$

$$6\sin^2 x - 8\sin x + 3\sin x - 4 = 0$$

$$2\sin x(3\sin x - 4) + 3\sin x - 4 = 0$$

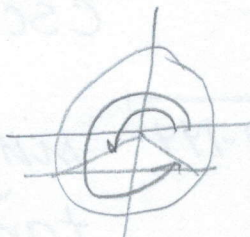
$$(3\sin x - 4)(2\sin x + 1) = 0$$

$$\sin x = \frac{4}{3} - \text{no sol.}$$

$$\sin x = -\frac{1}{2}$$

$$x = \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}$$

$$x_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \checkmark; x_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \checkmark$$



$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

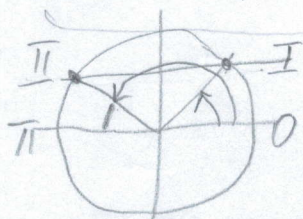
6) Solve $4\sin^2(2x) - 3 = 0, 0 \leq x \leq 2\pi$

[T5]

$$4\sin^2 2x = 3$$

$$\sin(2x) = \pm \frac{\sqrt{3}}{2}$$

$$\sin 2x = \frac{\sqrt{3}}{2}$$



$$T = \frac{2\pi}{2} = \pi = \frac{6\pi}{6}$$

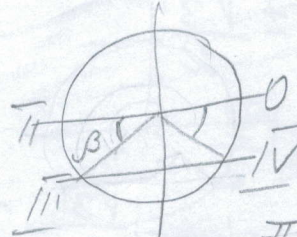
$$2x = \frac{\pi}{3}; x = \frac{\pi}{6} \checkmark; 2x = \frac{2\pi}{3}; x = \frac{\pi}{3} \checkmark$$

$$x_1 = \frac{\pi}{6}$$

$$x_2 = \frac{\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

$$\sin 2x = -\frac{\sqrt{3}}{2}$$



$$2x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$x_3 = \frac{2\pi}{3}$$

$$2x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

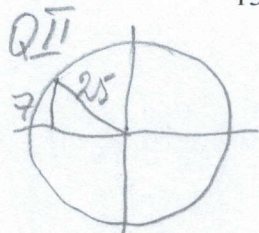
$$x_4 = \frac{5\pi}{6}$$

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- 7) Determine the **exact values** of $\csc 2x$ and $\tan(x-y)$ given $\sin x = \frac{7}{25}, \frac{\pi}{2} \leq x \leq \pi$ and

$\cos y = \frac{12}{13}, \frac{3\pi}{2} \leq y \leq 2\pi$. Include well-labelled diagrams.

[A4, C2]

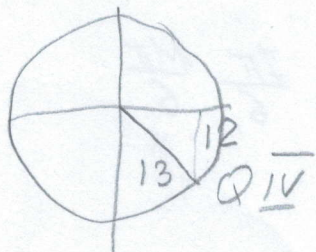


$$x = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \pm 24$$

$$\cos x = -\frac{24}{25}$$

$$\sin 2x = 2 \cdot \frac{7}{25} \cdot \left(-\frac{24}{25}\right) = -\frac{336}{625}$$

$$\csc 2x = \left(-\frac{625}{336}\right)$$



$$y = \sqrt{13^2 - 12^2} = \pm 5$$

$$\sin y = -\frac{5}{13}$$

$$\tan y = -\frac{5}{13} \times \frac{13}{12} = -\frac{5}{12}$$

$$\tan x = -\frac{7}{25} \div \frac{24}{25} = -\frac{7}{24}$$

$$\tan(x-y) =$$

- 8) Why do the solution(s) to the equation $\cos^2 x = \frac{3}{4}$ lie in all four quadrants? **DO NOT SOLVE**. Explain your answer.

[C2, T2]

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

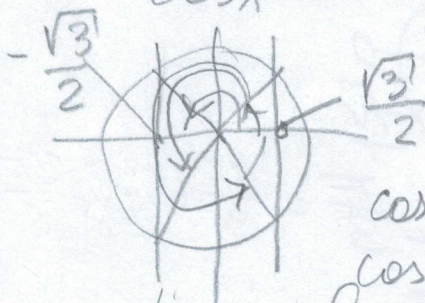
$$= \frac{-\frac{7}{24} + \frac{5}{12}}{1 + \frac{7}{24} \cdot \frac{5}{12}}$$

$$= \frac{-7 + 10}{24}$$

$$= \frac{3}{24} \times \frac{24 \cdot 12}{323}$$

$$= \frac{36}{323}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$



$$\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

4x values
in each quadrant